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Bianchi Type V Barotropic Perfect Fluid Cosmological Model in Lyra Geometry

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Abstract We have investigated Bianchi Type V barotropic perfect fluid cosmological model in Lyra geometry. To get the deterministic model of the universe, we have assumed the barotropic perfect fluid condition $p = \gamma\rho$, $0 \leq \gamma \leq 1$ and energy conservation equation i.e. $T^j_{i;j} = 0$. The physical and geometrical aspects of the model are discussed. The special cases for $\gamma = 1$ (stiff fluid distribution), $\gamma = 0$ (dust distribution), $\gamma = 1/3$ (disordered radiation) are also discussed.

Keywords Bianchi V · Barotropic · Lyra

1 Introduction

Einstein introduced his General Theory of Relativity in which gravitation is described in terms of geometry of space time and it motivated him to geometrize other physical fields. Weyl [1] made one of the best attempts in this direction. He introduced a generalization of Riemannian geometry in an attempt to unify gravitation and electromagnetism. Weyl's theory was not taken seriously because it was based on the non-integrability of length transfer. Later Lyra [2] suggested a modification of Riemannian geometry which has a close resemblance to Weyl's geometry. In Lyra's geometry, the connection is metric preserving as in Riemannian geometry and length transfer is integrable. Lyra introduced a gauge function which removed the non-integrability condition of the length of a vector under parallel transport. Thus Riemannian geometry was modified by Lyra and was given a new name called Lyra's geometry. Halford [3] in his study has shown that the constant displacement vector field in Lyra geometry plays the role of cosmological constant in General Relativity.

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Soleng [4] investigated cosmological models based on Lyra geometry and pointed out that the displacement field includes either a creation field and is equal to Hoyle-Narlikar creation field [5, 6] or contains a special vacuum field which with a gauge vector can be considered as a cosmological term. Singh and Singh have [7] given a review on Lyra geometry. The cosmological models based on Lyra geometry have been investigated by number of authors viz. Reddy and Venkateshwarlu [8], Singh and Singh [9], Chakravorty and Ghosh [10], Rahaman and Bera [11], Rahaman et al. [12, 13], Pradhan and Vishwakarma [14], Pradhan et al. [15, 16], Casana et al. [17], Singh [18], Kumar and Singh [19], Pradhan [20, 21], Mohanty et al. [22], Bali and Chandnani [23, 24].

Spatially homogeneous Bianchi Type V cosmological models create more interest in the study because of richer structure both physically and geometrically than the standard perfect fluid Friedmann-Robertson-Walker (FRW) models. These models are the simple generalization of the negative curvature of FRW models. A number of authors have studied Bianchi Type V models viz. Beesham [25], Maharaj and Beesham [26], Coley [27], Lorentz [28], Roy and Singh [29], Banerjee and Sanyal [30], Roy and Prasad [31], Nayak and Sahoo [32], Bali and Singh [33], Bali and Sharma [34], Bali and Meena [35, 36], Singh et al. [37], Bali and Jain [38]. Singh and Singh [39] have investigated some Bianchi Type V and VI_0 cosmological models in Lyra geometry. To get the deterministic models of the universe, they have discussed a stiff perfect fluid $p=\rho$ model where p the isotropic pressure and ρ the matter density. In both the models, the gauge function β is time dependent and $\beta=$ constant, are considered.

In this paper, we have investigated Bianchi Type V barotropic perfect fluid cosmological model in Lyra geometry. To get the deterministic model of the universe, we have assumed the barotropic perfect fluid condition $p=\gamma\rho$, $0\le\gamma\le 1$ and energy conservation equation i.e. $T^{j}_{i;j}=0$. The physical and geometrical aspects of the model are discussed. The special cases for $\gamma=1$ (stiff fluid distribution), $\gamma=0$ (dust distribution), $\gamma=1/3$ (disordered radiation) are also discussed.

2 The Metric and Field Equations

We consider Bianchi type V metric in the form

$$ds^2=-dt^2+A^2dx^2+B^2e^{2x}dy^2+C^2e^{2x}dz^2 \tag{2.1}$$

where A, B, C are functions of t-alone.

Einstein's field equations in normal gauge for Lyra's manifold obtained by Sen [40] is given by

$$R^{j}_{i}-\frac{1}{2}Rg^{j}_{i}+\frac{3}{2}\phi_i\phi^j-\frac{3}{4}\phi_k\phi^k g^{j}_{i}=-\frac{8\pi G}{c^4}T^{j}_{i} \tag{2.2}$$

Energy momentum tensor T^{j}_{i} for perfect fluid distribution is given by

$$T^{j}_{i}=(\rho+p)v_i v^j+pg^{j}_{i} \tag{2.3}$$

where $v_i=(0,0,0,-1)$; $v^i v_i=-1$; $\phi_i=(0,0,0,\beta(t))$; $v_4=-1$ and $v^4=1$. p is the isotropic pressure, ρ the matter density, v^i the fluid flow vector and β the gauge function.

The field equations for the metric (2.1) lead to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-p \tag{2.4}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-p \tag{2.5}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}+\frac{3}{4}\beta^2=-p \tag{2.6}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}-\frac{3}{A^2}-\frac{3}{4}\beta^2=\rho \tag{2.7}$$

$$\frac{2A_4}{A}-\frac{B_4}{B}-\frac{C_4}{C}=0 \tag{2.8}$$

Here we have used the geometrized unit in which $8\pi G=1$, $c=1$.

The energy conservation equation $T^j_{i;j}=0$ leads to

$$\rho_4+(\rho+p)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \tag{2.9}$$

and the conservation of left hand side of (2.2) leads to

$$\left(R^j_i-\frac{1}{2}Rg^j_i\right)_{;j}+\frac{3}{2}(\phi_i\phi^j)_{;j}-\frac{3}{4}(\phi_k\phi^k g^j_i)_{;j}=0 \tag{2.10}$$

Equation (2.10) leads to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j}+\phi^\ell\Gamma^j_{\ell j}\right]+\frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j}-\phi_\ell\Gamma^\ell_{ij}\right]-\frac{3}{4}g^j_i\phi_k\left[\frac{\partial\phi^k}{\partial x^j}+\phi^\ell\Gamma^k_{\ell j}\right]$$
$$-\frac{3}{4}g^j_i\phi^k\left[\frac{\partial\phi_k}{\partial x^j}-\phi_\ell\Gamma^\ell_{kj}\right]=0 \tag{2.11}$$

Equation (2.11) is automatically satisfied for $i=1,2,3$.

For $i=4$, (2.11) leads to

$$\frac{3}{2}\phi_4\left[\frac{\partial}{\partial x^4}(g^{44}\phi_4)+\phi^4\Gamma^j_{4j}\right]+\frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial x^4}-\phi_4\Gamma^4_{44}\right]$$
$$-\frac{3}{4}g^4_4\phi_4\left[\frac{\partial\phi_4}{\partial x^4}+\phi^4\Gamma^4_{44}\right]-\frac{3}{4}g^4_4g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial x^4}-\phi^4\Gamma^4_{44}\right]=0 \tag{2.12}$$

which again leads to

$$\frac{3}{2}\beta\beta_4+\frac{3}{2}\beta^2\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \tag{2.13}$$

3 Solution of Field Equations

From (2.4) and (2.5), we have

$$\frac{B_{44}}{B}+\frac{B_4C_4}{BC}=\frac{A_{44}}{A}+\frac{A_4C_4}{AC} \tag{3.1}$$

Equations (2.5) and (2.6) lead to

$$\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=\frac{B_{44}}{B}+\frac{A_4B_4}{AB} \tag{3.2}$$

From (2.8), we have

$$\frac{2A_4}{A}=\frac{B_4}{B}+\frac{C_4}{C} \tag{3.3}$$

which leads to

$$A=\ell(BC)^{1/2} \tag{3.4}$$

ℓ being constant of integration.

Equation (2.8) also leads to

$$\frac{A_4}{A}=\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right) \tag{3.5}$$

From (3.5), we have

$$\frac{A_{44}}{A}=\frac{1}{2}\left(\frac{B_{44}}{B}+\frac{C_{44}}{C}\right)-\frac{1}{4}\left(\frac{B_4^2}{B^2}+\frac{C_4^2}{C^2}\right)+\frac{B_4C_4}{2BC} \tag{3.6}$$

Multiplying (2.7) by γ and adding into (2.6), we have

$$\begin{aligned}&\frac{A_{44}}{A}+\frac{B_{44}}{B}+(1+\gamma)\frac{A_4B_4}{AB}-(1+3\gamma)\frac{1}{A^2}+\frac{3(1-\gamma)}{4}\beta^2\\&\quad+\frac{\gamma B_4C_4}{BC}+\frac{\gamma A_4C_4}{AC}=\gamma\rho-p\end{aligned} \tag{3.7}$$

Applying the Barotropic fluid condition, i.e. $p=\gamma\rho$, (3.7) leads to

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+(1+\gamma)\frac{A_4B_4}{AB}-(3\gamma+1)\frac{1}{A^2}+\frac{3(1-\gamma)}{4}\beta^2+\gamma\frac{B_4C_4}{BC}+\gamma\frac{A_4C_4}{AC}=0 \tag{3.8}$$

Using (3.5), in (2.13), we have

$$\beta=\frac{W}{(BC)^{3/2}} \tag{3.9}$$

W being the constant of integration.

Now using (3.5), (3.6) and (3.9), equation (3.8) leads to

$$\begin{aligned}&\frac{3}{2}\frac{B_{44}}{B}+\frac{1}{2}\frac{C_{44}}{C}+(2\gamma+1)\frac{B_4C_4}{BC}+\left(\frac{2\gamma+1}{4}\right)\frac{B_4^2}{B^2}+\left(\frac{2\gamma-1}{4}\right)\frac{C_4^2}{C^2}\\&\quad-\frac{(3\gamma+1)}{\ell^2BC}-\frac{3(\gamma-1)W^2}{4(BC)^3}=0\end{aligned} \tag{3.10}$$

We assume that

$$BC=\mu \tag{3.11}$$

$$\frac{B}{C}=\nu \tag{3.12}$$

Hence

$$\frac{B_4}{B}=\frac{1}{2}\left(\frac{\mu_4}{\mu}+\frac{\nu_4}{\nu}\right) \tag{3.13}$$

$$\frac{C_4}{C}=\frac{1}{2}\left(\frac{\mu_4}{\mu}-\frac{\nu_4}{\nu}\right) \tag{3.14}$$

$$\frac{B_{44}}{B}=\frac{1}{2}\left(\frac{\mu_{44}}{\mu}+\frac{\nu_{44}}{\nu}+\frac{\mu_4\nu_4}{\mu\nu}\right)-\frac{1}{4}\left(\frac{\mu_2^2}{\mu^2}+\frac{\nu_4^2}{\nu^2}\right) \tag{3.15}$$

$$\frac{C_{44}}{C}=\frac{1}{2}\left(\frac{\mu_{44}}{\mu}-\frac{\nu_{44}}{\nu}-\frac{\mu_4\nu_4}{\mu\nu}\right)+\frac{1}{4}\left(\frac{3\nu_4^2}{\nu^2}-\frac{\mu_4^2}{\mu^2}\right) \tag{3.16}$$

Using (3.11)–(3.16) in (3.10), we have

$$\frac{\mu_{44}}{\mu}+\left(\frac{3\gamma-1}{4}\right)\frac{\mu_4^2}{\mu^2}+\frac{1}{2}\frac{\nu_{44}}{\nu}+\frac{3}{4}\frac{\mu_4\nu_4}{\mu\nu}-\left(\frac{\gamma+1}{4}\right)\frac{\nu_4^2}{\nu^2}$$
$$-\frac{(3\gamma+1)}{\ell^2}\frac{1}{\mu}-\frac{3}{4}\frac{(\gamma-1)W^2}{\mu^3}=0 \tag{3.17}$$

Also (3.2) and (3.3) lead to

$$\frac{(CB_4-BC_4)_4}{CB_4-BC_4}=-\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right) \tag{3.18}$$

which leads to

$$C^2\left(\frac{B}{C}\right)_4=\frac{L}{\sqrt{BC}} \tag{3.19}$$

Using (3.11) and (3.12), equation (3.19) leads to

$$\frac{\nu_4}{\nu}=\frac{L}{\mu^{3/2}} \tag{3.20}$$

L being constant of integration.

Using (3.20) in (3.17), we have

$$\frac{\mu_{44}}{\mu}+\left(\frac{3\gamma-1}{4}\right)\frac{\mu_4^2}{\mu^2}+\frac{1}{2}\left(-\frac{3L}{2\mu^{3/2}}\frac{\mu_4}{\mu}+\frac{L^2}{\mu^3}\right)+\frac{3}{4}\frac{L}{\mu^{3/2}}\frac{\mu_4}{\mu}-\frac{(1+\gamma)}{4}\frac{L^2}{\mu^3}$$
$$-\left(\frac{3\gamma+1}{\ell^2}\right)\frac{1}{\mu}-\frac{3}{4}\frac{(\gamma-1)W^2}{\mu^3}=0 \tag{3.21}$$

which leads to

$$2\mu_{44}+\left(\frac{3\gamma-1}{2}\right)\frac{\mu_4^2}{\mu}=\frac{(\gamma-1)}{2}(3W^2+L^2)\frac{1}{\mu^2}+\frac{2}{\ell^2}(3\gamma+1) \tag{3.22}$$

Now we assume

$$\mu_4=f(\mu) \tag{3.23}$$

Therefore

$$\mu_{44} = ff'$$

where

$$f' = df/d\mu \tag{3.24}$$

Using (3.23) and (3.24) into (3.22), we have

$$\frac{d}{d\mu}(f^2) + \frac{\alpha}{\mu} f^2 = \frac{b}{\mu^2} + N \tag{3.25}$$

where

$$\alpha = \left(\frac{3\gamma - 1}{2}\right)$$

$$b = \left(\frac{\gamma - 1}{2}\right)(3W^2 + L^2)$$

$$N = \frac{2}{\ell^2}(3\gamma + 1)$$

Now (3.25) leads to

$$f^2 = \left(\frac{3W^2 + L^2}{3}\right)\frac{1}{\mu} + \frac{4}{\ell^2}\mu + \frac{S}{\mu^\alpha} \tag{3.26}$$

where S is constant of integration.

Equation (3.26) leads to

$$\frac{d\mu}{dt} = f = \left[\left(\frac{3W^2 + L^2}{3}\right)\frac{1}{\mu} + \frac{4}{\ell^2}\mu + \frac{S}{\mu^\alpha}\right]^{1/2} \tag{3.27}$$

Now (3.20) leads to

$$\frac{d\nu}{\nu} = \frac{L}{\mu^{3/2}} dt = \frac{L}{\mu^{3/2}}\left(\frac{dt}{d\mu}\right) d\mu$$

which leads to

$$\log \nu = \int \frac{L}{\mu^{3/2}} \frac{1}{\sqrt{(\frac{3W^2+L^2}{3})\frac{1}{\mu} + \frac{4}{\ell^2}\mu + \frac{S}{\mu^\alpha}}} d\mu \tag{3.28}$$

Hence the metric (2.1) leads to

$$ds^2 = \frac{-1}{[(\frac{3W^2+L^2}{3})\frac{1}{T} + \frac{4}{\ell^2}T + \frac{S}{T^\alpha}]} dT^2 + \ell^2 T dx^2 + T\nu e^{2x} dy^2 + \frac{T}{\nu} e^{2x} dz^2 \tag{3.29}$$

where cosmic time t is given by (3.27) as

$$t = \int \frac{dT}{\sqrt{(\frac{3W^2+L^2}{3})\frac{L}{T} + \frac{4T}{\ell^2} + \frac{S}{T^\alpha}}} \tag{3.30}$$

where ν is determined by (3.28) and $\mu = T$.

4 Some Physical and Geometrical Features

The displacement vector (β) is given by (3.9) as

$$\beta = \frac{W}{(BC)^{3/2}}$$

which leads to

$$\beta = \frac{W}{T^{3/2}} \tag{4.1}$$

where $BC = \mu$ and $\mu = T$.

The expansion (θ) is given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

which leads to

$$\begin{aligned}\theta &= \frac{3}{2T}\left[\left(\frac{3W^2 + L^2}{3}\right)\frac{1}{T} + \frac{4}{\ell^2}T + \frac{S}{T^\alpha}\right]^{1/2} \\ &= \frac{3}{2T^{1+\frac{\alpha}{2}}}\left[\left(\frac{3W^2 + L^2}{3}\right)T^{\alpha-1} + \frac{4}{\ell^2}T^{\alpha+1} + S\right]\end{aligned} \tag{4.2}$$

The components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3}\left(\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \tag{4.3a}$$

$$\begin{aligned}\sigma_2^2 &= \frac{1}{3}\left(\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right) \\ &= \frac{L}{2T^{3/2}}\end{aligned} \tag{4.3b}$$

$$\begin{aligned}\sigma_3^3 &= \frac{1}{3}\left(\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right) \\ &= -\frac{L}{2T^{3/2}}\end{aligned} \tag{4.3c}$$

$$\sigma_4^4 = 0 \tag{4.3d}$$

Hence the shear (σ) is given by

$$\begin{aligned}\sigma^2 &= \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^3 + (\sigma_4^4)^4] \\ \sigma &= \frac{L}{2T^{3/2}}\end{aligned} \tag{4.4}$$

From (2.7), matter density (ρ) is given by

$$\rho = \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{3}{A^2} - \frac{3}{4}\beta^2$$

which leads to

$$\rho = -\frac{3}{4}\beta^2 + \frac{3}{4}\left(\frac{\mu_4}{\mu}\right)^2 - \frac{L^2}{4\mu^3} - \frac{3}{\ell^2\mu}$$

which again leads to

$$\rho = -\frac{3}{4}\beta^2 + \frac{3}{4}\frac{W^2}{T^3} + \frac{S}{T^{\alpha+2}} \tag{4.5}$$

Isotropic pressure (p) is given by

$$\begin{aligned} p &= \gamma\rho \\ &= \gamma\left[-\frac{3}{4}\beta^2 + \frac{3}{4}\frac{W^2}{T^3} + \frac{S}{T^{\alpha+2}}\right] \end{aligned} \tag{4.6}$$

The spatial volume (R^3) is given by

$$R^3 = \ell T^{3/2} e^{2x} \tag{4.7}$$

The deceleration parameter (q) is given by

$$q = -\frac{R_{44}/R}{R_4^2/R^2}$$

which leads to

$$q = \frac{\frac{2}{3T}(3W^2 + L^2) + (\frac{3\gamma+1}{2})\frac{S}{T^\alpha}}{(\frac{3W^2+L^2}{3})\frac{1}{T} + \frac{4T}{\ell^2} + \frac{S}{T^\alpha}} \tag{4.8}$$

5 Special Case

5.1 Stiff Fluid Model i.e. $\gamma = 1$

Taking $\gamma = 1$ into (3.27), we have

$$f = \left[\left(\frac{3W^2 + L^2 + 3S}{3}\right)\frac{1}{T} + \frac{4}{\ell^2}T\right]^{1/2} \tag{5.1}$$

For $\gamma = 1$, (3.28) leads to

$$\log \nu = \frac{\ell L}{2}\int \frac{dT}{T\sqrt{(\frac{3W^2+L^2+3S}{3})\frac{\ell^2}{4} + T^2}}$$

which leads to

$$\log \nu = \frac{\ell L}{2}\int \frac{dT}{T\sqrt{T^2 + a^2}} \tag{5.2}$$

where

$$a^2 = \frac{(3W^2 + L^2 + 3S)\ell^2}{12}$$

Equation (5.2) leads to

$$\nu = M\left[\frac{a-\sqrt{a^2+T^2)}}{T}\right]^{\frac{L\ell}{2a}} \tag{5.3}$$

where M is a constant.

In this case, the metric (2.1) leads to

$$ds^2 = -\frac{T\ell^2 dT^2}{4[T^2+a^2]} + \ell^2 T dx^2 + TM\left[\frac{a-\sqrt{a^2+T^2}}{T}\right]^{\frac{L\ell}{2a}} e^{2x}dy^2 + \frac{T}{M}\left[\frac{a-\sqrt{a^2+T^2}}{T}\right]^{-\frac{L\ell}{2a}} e^{2x}dz^2 \tag{5.4}$$

The displacement vector β is given by

$$\beta = \frac{W}{T^{3/2}} \tag{5.5}$$

The expansion (θ) is given by

$$\theta = \frac{3\ell}{T^{3/2}}[T^2+a^2]^{1/2} \tag{5.6}$$

The components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = 0 \tag{5.7a}$$

$$\sigma_2^2 = \frac{L}{2T^{3/2}} \tag{5.7b}$$

$$\sigma_3^3 = -\frac{L}{2T^{3/2}} \tag{5.7c}$$

$$\sigma_4^4 = 0 \tag{5.7d}$$

Hence the shear (σ) is given by

$$\sigma = \frac{L}{2T^{3/2}} \tag{5.8}$$

Matter density (ρ) and *isotropic pressure* (p) are given by

$$\rho = p = -\frac{3}{4}\beta^2 + \left(\frac{3}{4}w^2 + S\right)\frac{1}{T^3} \tag{5.9}$$

The *deceleration parameter* (q) is given by

$$q = \frac{2(3W^2+L^2)+6S}{3[(\frac{3W^2+L^2}{3}) + \frac{4T^2}{\ell^2} + S]} \tag{5.10}$$

The *relative anisotropy* is given by

$$\frac{\sigma^2}{\rho} = \frac{L^2}{4S} > 0 \tag{5.11}$$

as $S > 0$.

Similarly, we obtain the same type of results for $\gamma = 0, 1/3$.

6 Discussion

For the model (3.29), the matter density $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$ where $\alpha + 2 > 0$. The reality condition $\rho > 0$ requires that $S > 0$. The model (3.29) starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases when $\alpha > 1$. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$. Hence anisotropy is maintained throughout. The spatial volume increases as time increases. We also find that $q > 0$, hence the model (3.29) represents a decelerating universe. The displacement vector $\beta \to 0$ when $T \to \infty$. When $T \to 0$ then $\beta \to \infty$. The model (3.29) has Point Type singularity at $T = 0$ (Mac Callum [41]).

For the model (5.4) (for $\gamma = 1$), the matter density $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$ where β is given by (5.5). The reality condition $\rho > 0$ requires that $S > 0$. The displacement vector $\beta \to 0$ when $T \to \infty$ and $\beta \to \infty$ when $T \to 0$. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \to 0$. Hence the model (5.4) isotropizes for large values of T. Since the deceleration parameter $q > 0$, hence the model (5.4) represents a decelerating universe. The model (5.4) has Point Type singularity at $T = 0$. The model (5.4) obtained for stiff perfect fluid is the same as obtained by Singh and Singh [39].

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